Total number of printed pages–7

3 (Sem–5/CBCS) PHY HC1

2024

PHYSICS

(Honours Core)

Paper : PHY–HC–5016

(Quantum Mechanics and Applications)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×7=7

(a) Eigenvalue of Hamiltonian operator is

(i) kinetic energy

(ii) potential energy

(iii) both (i) and (ii)

(iv) total energy

(b) Why $\psi = e^{x}$ is not an acceptable wave function in quantum mechanics ?

Contd.

(c) What do you mean by space quantisation of an atom ?

(d) The value of $\left[\hat{x}, \frac{\partial}{\partial x}\right]$ is

(i) 1

(ii) -1

(iii) $i\hbar$

(iv) $-i$

(e) What is the value of spin-orbit interaction energy for the ground state of hydrogen atom ?

(f) When does the probability density of a quantum mechanical oscillator approach that of a classical oscillator ?

(g) Can the Stern-Gerlach experiment be performed with ions instead of neutral atoms ?

2. Answer the following questions : 2×4=8

(a) Is the wave function $\psi(x) = e^{ikx}$ an eigenfunction of the kinetic energy operator T ? If yes, what is its eigenvalue ?

(b) What is a Gaussian wave packet ? Express its wave function.

(c) The one-dimensional wave function is given by $\psi(x) = \sqrt{a}\, e^{-ax}$. Find the probability of finding the particle between $x = \frac{1}{a}$ and $x = \frac{2}{a}$.

(d) Calculate the Lande's g factor for the $^{2}P_{3/2}$ state.

3. Answer ***any three*** of the following questions : 5×3=15

(a) State the conditions of "acceptability of wave function" in quantum mechanics with explanation.

(b) Obtain time-independent Schrödinger wave equation for a free particle in one dimension. Give a physical interpretation of the wave function $\psi(x,t)$. 4+1=5

Contd.

(c) Find the expectation value of energy when the state of harmonic oscillator is described by the following wave function :

$$\psi(x,t) = \frac{1}{\sqrt{2}}\left[\psi_0(x,t) + \psi_1(x,t)\right]$$

where $\psi_0(x,t)$ and $\psi_1(x,t)$ are the wave functions for the ground state and first excited state respectively.

(d) State Pauli's exclusion principle. An atomic state is denoted by $^3p_2$. Determine the values of L, S and J and mention whether the above state is admissible or not. 2+3=5

(e) Discuss the significance of zero-point energy with reference to a linear harmonic oscillator. The energy of a linear harmonic oscillator in the third excited state is 0.1 eV. Find the frequency of vibration. 2+3=5

4. Answer ***any three*** of the following questions : 10×3=30

(a) (i) What is the need for normalization of a wave function ? A wave function $\psi(x)$ is given by

$$\psi(x) = A_n \sin\frac{2n\pi x}{L}$$ in the region $0 \le x \le L$. Find the value of A_n using normalization condition. 1+4=5

(ii) Derive the continuity equation from the time-dependent Schrödinger equation of a particle moving in a real potential and give its physical significance. 4+1=5

(b) A particle of mass m is moving in a one-dimensional potential given by

$v(x) = 0$ for $0 \le x \le L$

$v(x) = \infty$ for $x < 0$ and $x > L$

Using appropriate boundary conditions, solve the Schrödinger equation and find allowed energy values and normalized wave functions of the particle. Also plot the eigenfunctions corresponding to different eigenvalues. 8+2=10

(c) Write the radial equation of hydrogen atom and solve it for obtaining its energy eigenvalues. 2+8=10

(d) What is anomalous Zeeman effect ? Discuss the quantum mechanical theory of anomalous Zeeman effect, with special reference to Zeeman pattern for D_1 and D_2 lines of sodium. 2+8=10

(e) (i) Describe and explain LS and JJ couplings. Illustrate them with vector diagram. 2+2+4=8

(ii) Determine the possible values of resultant angular momentum for two electrons having $j_1 = \frac{3}{2}$ and $j_2 = \frac{5}{2}$. 2

(f) (i) A beam of electrons enters a uniform magnetic field of flux density $1.2 Wb/m^2$ in the z-direction. Find the energy difference between the electrons whose spins are parallel and anti-parallel to the field. 5

(ii) Write short note on ***any one*** of the following : 5

(i) Paschen-Back effect

(ii) Stark effect

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