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3 (Sem–4/CBCS) MAT HC 2

2024

MATHEMATICS

(Honours Core)

Paper : MAT–HC–4026

(Numerical Methods)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : 1×7=7

 (a) Name the three basic components of an algorithm.

 (b) Show $\nabla E \equiv \Delta$.

 (c) Write down the Lagrangian linear interpolation formula at the points x_0 and x_1 with corresponding function values f_0 and f_1.

Contd.

 (d) What is the order of convergence of secant method ?

 (e) The approximation formula for finding the derivative at x_0 given by

$$f'(x_0) = \frac{f(x_0+h) - f(x_0)}{h} - \frac{h}{2} f''(\xi), \quad x_0 < \xi < x_{0+h}$$

 is a

 (i) backward difference approximation formula of first order of approximation

 (ii) forward difference approximation formula of second order of approximation

 (iii) forward difference approximation formula of first order of approximation

 (iv) None of the above

 (Choose the correct option)

 (f) What is numerical integration ? What is its general form ?

 (g) Name a method for approximating a solution to an initial value problem.

2. Answer the following questions : 2×4=8

 (a) Compute the following limit and determine the rate of convergence

$$\lim_{x \to 0} \frac{e^x - 1}{x}.$$

 (b) Prove $(I + \Delta)(I - \nabla) \equiv I$.

 (c) Show that LU decomposition of a matrix is unique up to scaling by a diagonal matrix.

 (d) Find the approximate value of $\int_0^1 \frac{dx}{1+x}$ by Simpson's rule.

3. Answer ***any three*** : 5×3=15

 (a) Construct an iteration function corresponding to the given function $f(x) = x^3 - x^2 - 10x + 7$.

 Use the fixed point iteration scheme with initial approximation as $P_0 = 1$ and perform three iterations to approximate the root of $f(x) = 0$.

(b) Using the data given below form the divided difference table and use it to construct the Newton form of the interpolating polynomial :

x	−1	0	1	2
y	5	1	1	11

(c) Use four iterations of Newton's method to approximate the root of the equation

$$f(x) = x^3 + 2x^2 - 3x - 1$$

in the interval (1, 2) starting with an initial approximation of $P_0 = 1$.

(d) Derive the second order central difference approximation for first derivative including error term given by

$$f'(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h} - \frac{h^2}{6} f'''(\xi)$$

(e) (i) Name the measures by which errors are quantified. Write down the expressions for the same.

(ii) Prove that $\Delta^n f(x_i) = (E - I)^n f(x_i)$

4. Answer **any three** : 10×3=30

(a) What is Theoretical Error Bound ? Show that the Bisection Method for approximating a root of the equation $f(x) = 0$ always converges. Find the order of convergence of the Bisection Method. 1+6+3=10

(b) Verify that the equation $x^3 + x^2 - 3x - 3 = 0$ has a root in the interval (1, 2). Given that the exact root is $x = \sqrt{3}$, perform the first three iterations of the Regula-Falsi method. What is the computable estimate for $|e_n|$, the error obtained in nth step by this method. Verify that the absolute error in the third approximation satisfies the error estimate. 1+6+3=10

(c) What is an interpolating polynomial? Determine the interpolation error when a function is approximated by a constant polynomial. Mention an advantage and a disadvantage of Lagrangian form of the interpolating polynomial. Derive the Lagrangian interpolating polynomial for the given data : 1+2+2+5=10

x	−2	−1	0	1	2	3
y	39	3	−1	−3	−9	−1

(d) What are two different classes of methods for solving a linear system of equations. Name one method of each type. What do you mean by an LU decomposition of square matrix A.

Solve the following system using LU decomposition : 1+1+8=10

$$2x_1 + 7x_2 + 5x_3 = -4$$
$$6x_1 + 20x_2 + 10x_3 = -16$$
$$4x_1 + 3x_2 = -7$$

(e) (i) Derive the basic Trapezoidal rule for integrating $\int_a^b f(x)dx$. 6

(ii) Use appropriate first order approximation formulas to find derivatives of the values of $f(x)$ at the points $x = 0{\cdot}5$, $x = 0{\cdot}6$ and $x = 0{\cdot}7$. 4

x	$f(x)$	$f'(x)$
0·5	0·4794	?
0·6	0·5646	?
0·7	0·6442	?

(f) What is the basic problem that is solved by Euler's method ? Derive Euler's method. Given that the exact solution to $\frac{dx}{dt} = \frac{t}{x}$ is $x(t) = \sqrt{t^2+1}$, find the absolute error at each step that is obtained by solving

$$\frac{dx}{dt} = \frac{t}{x},\ 0 \le t \le 1{\cdot}0,\ x(0) = 1,\ h = 0{\cdot}5$$

by Euler's method. 1+4+5=10