Dedicated to my loving parents Hemanta Kumar Sarma and Hemasri Sarma…

DECLARATION

I hereby declare that this Ph.D. thesis entitled "**Exact Solutions of Some Hydromagnetic and Hydrodynamic Convective Flow Problems**" is the result of my own research work done under the guidance and supervision of Dr. Nazibuddin Ahmed, Professor and Head, Department of Mathematics, Gauhati University, Guwahati. Further, I also declare that neither this thesis nor any part of it has been utilized or submitted to any other University or Institution for the perusal of any other degree or diploma.

This thesis contains less than 90,000 (ninety thousand) words excluding bibliography and captions.

Date:
Place: Guwahati, Assam

Subhrajit Sarma
Assistant Professor
Department of Mathematics,
G.L.C. College, Barpeta Road

Dr. Nazibuddin Ahmed
Professor and Head,
Department of Mathematics,
Gauhati University, Guwahati-14,
Assam, India.
E-mail: nazib@gauhati.ac.in

CERTIFICATE

This is to certify that the thesis entitled "**Exact Solutions of Some Hydromagnetic and Hydrodynamic Convective Flow Problems**" which is being submitted to Gauhati University, by Subhrajit Sarma, in partial fulfilment of the requirements for the award of the Degree of Doctor of Philosophy, is the result of his research work carried out under my supervision. No part of this work was previously submitted to elsewhere for the award of any other degree. He fulfils all the requirements as per the regulations for the award of the degree of Doctor of Philosophy of Gauhati University, Guwahati.

(Prof. Nazibuddin Ahmed)

Date:

Place: Guwahati, Assam

ACKNOWLEDGEMENT

First and foremost, I express my deep sense of gratitude towards my supervisor Dr. Nazibuddin Ahmed, Professor and Head, Department of Mathematics, Gauhati University for the invaluable inspiration, advice, guidance and supervision I received from him at every stage of my research work.

I would remain sincerely thankful to Dr. Nilakshi Goswami, Associate Professor, Department of Mathematics, Gauhati University for supporting, inspiring and advising me during the period of my Ph.D. work. I am also thankful to all the faculty members of the Department of Mathematics, Gauhati University for offering valuable suggestions.

I am grateful to all the faculty members of my workplace G.L.C. College, Barpeta Road for their support. I am also thankful to the faculty members of Department of Mathematics, Tihu College for their constant encouragement and suggestions. I offer my acknowledgement for the helps provided by fellow research scholars Bijoy Krishna Taid, Nabanita Das, Rajdeep Bordoloi and Dr. Kalyan Chamuah.

I am profoundly thankful to my sister Dr. Riju Sarma, Assistant Professor, M.C. College, Barpeta for her constant motivation and encouragement.

I am forever indebted to my father Hemanta Kumar Sarma, my mother Hemasri Sarma and my brother in- law Bikash Sarma who have always inspired and stood by me.

I offer my regards and heartfelt gratitude to all the teachers in my life, friends, relatives, well- wishers and students who have encouraged me to pursue my Ph.D. research.

Last but not the least I would like to thank the Almighty for all that I have accomplished till now.

Date: (Subhrajit Sarma)

LIST OF PUBLICATIONS

1. Thermal diffusion effect on unsteady MHD free convective flow past an impulsively started but temporarily accelerated semi-infinite vertical plate with parabolic ramped conditions.
   Published in *Heat Transfer, Wiley* (**Scopus, ESCI**), **50**(4), 1-33, 2021.

2. Diffusion thermo effect on free convective flow past an impulsively started semi-infinite moving vertical plate with uniform heat and mass Flux.
   Published in *JP Journal of Heat and Mass Transfer* (**Scopus**), **24**(1), 37-62, 2021.
3. Dufour effect on unsteady MHD flow past a vertical plate embedded in porous medium with ramped temperature.
   Published in *Scientific Reports, Nature* (**Scopus, SCIE**), **12**(1), 2022.

4. Thermal diffusion effect on unsteady magnetohydrodynamic free convective, chemically reactive, heat absorbing and radiating flow past a semi-infinite moving vertical plate immersed in porous medium with arbitrary ramped temperature. Published in *Canadian Journal of Physics* (**Scopus, SCI**), **100**, 437-451, 2022.
5. Thermal diffusion and thermal radiation effects on free convective MHD radiating flow past an impulsively started infinite vertical plate.
   (In Press for publication in Proceeding in Springer)
6. Influence of thermal diffusion on unsteady MHD free convective, chemically reactive and radiating flow past a semi-infinite inclined moving plate in a porous medium with arbitrary ramped temperature.
   (Under communication).
7. Induced magnetic field and thermal diffusion effects on unsteady MHD free convective, chemically reactive and radiating flow past a semi-infinite moving vertical plate with arbitrary ramped temperature.
   (Under communication).

LIST OF CONFERENCES ATTENDED

1. International Conference on Applied Nonlinear Analysis and Soft Computing, 22-23 December, 2020, organized by Department of Mathematics, Gauhati University, Guwahati, Assam, India.
2. The 4^{th} International Conference on Mathematical Modelling, Applied Analysis and Computation-2021, 5-7 August, 2021, organized by Department of Mathematics, JECRC University, Jaipur, Rajasthan, India.

CONTENTS

Chapter IV

Chapter V

Chapter VI

Chapter VII

Chapter VIII

ABSTRACT

This thesis consists of eight chapters which are devoted to theoretical investigation of seven different heat and mass transfer problems under various physical conditions. The first chapter of the thesis is Introduction, and in the rest of the chapters contain one flow problem each. Except the problem in chapter IV, all other problems deal with MHD flows. While flow over inclined plate is considered only in Chapter VII, the other problems are confined to flow over vertical plate only. The flow geometries are one- dimensional and are accompanied by some simplifying assumptions on the continuity equation, energy equation, Navier-Stokes equations and the Species continuity equation. Except the density in the buoyancy force term, all other fluid properties are taken to be constant. In all the problems, flows are taken to be unsteady. All throughout, the solutions of all the problems are obtained by adopting a closed form of Laplace transform technique. The Joule heating effect as well as the viscous dissipations of energy has been neglected throughout the study. Effect of induced magnetic field is considered only in Chapter II and Chapter VIII. Medium of flow is taken to be porous in chapter V, chapter VI and chapter VII. With the exception of the fourth chapter, the remaining chapters of the present work deal with radiation effect. Effect of chemical reaction has been considered in chapter II, chapter V, chapter VI, chapter VII and chapter VIII. In chapter II and chapter VI, heat absorption effect is taken onto account. Thermal diffusion effect is considered in second, third, sixth, seventh and eighth chapters while diffusion thermo effect is considered in fourth and fifth chapters. Furthermore condition of parabolic ramped temperature and concentration is taken in chapter III whereas arbitrary ramped temperature condition is studied in chapters from V to VIII.

Chapter I present a comprehensive discussion of the subject matter relevant to the thesis. Basic concepts related to the mechanisms of heat and mass transfer are discussed. A brief history of the development of the subject and review of the existing literature also included in this chapter.

In **Chapter II**, an exact solution to the problem of a free convective MHD radiating chemically reacting non-gray optically thick flow of incompressible fluid past a suddenly started semi-infinite vertical plate in presence of a transverse magnetic field, taking into account the effects of thermal diffusion, and induced magnetic field, is obtained. A closed-

form Laplace transform technique is adopted to derive the solutions of the governing equations. The Rosseland approximation is utilized to describe the heat flux due to radiation that appears in the energy equation. The effects of magnetic field, thermal radiation, thermal diffusion, chemical reaction on the flow and transport characteristics are merely focused in the study. It is seen that the induced magnetic field gets enhanced for high magnetic diffusivity. Further, the fluid temperature rises significantly as time progresses.

The purpose of **Chapter III** is to analyse the problem of a free convective MHD flow of incompressible, electrically conducting and viscous fluid past an impulsively started semi-infinite moving vertical plate. The fluid is considered to be non- gray and optically thick. The parabolic ramped temperature of the plate and thermo diffusion effect is also taken onto account. A magnetic field having uniform strength is applied at transverse direction to the fluid velocity. Solutions of dimensionless governing partial differential equations are attained on adoption of closed form Laplace transformation technique. Effect of different flow parameters on velocity field, temperature field, concentration field, Nusselt number, skin friction and Sherwood Number are discussed graphically. It is noticed that fluid concentration, temperature and velocity declines considerably for ascending values of Prandtl Number. Increasing Ramped parameter hikes Nusselt number and Sherwood Number but declines skin friction.

The aim of **Chapter IV** is to study analytically the problem of free convective, unsteady hydrodynamic flow of a viscous and incompressible fluid past a semi- infinite vertical plate with uniform heat and mass transfer in presence of diffusion thermo effect. Exact solutions of the dimensionless governing partial differential equations are obtained by adopting closed form Laplace transformation technique. Expressions of concentration field, temperature field, velocity field, plate concentration, plate temperature, Nusselt number, Sherwood number and skin friction are attained and effect of various flow parameters on them are displayed graphically. The study reveals that Dufour effect accelerates both fluid temperature and plate temperature.

Chapter V aims to find an exact solution to the problem of a free convective, viscous, radiating, chemically reacting, optically thick, non-gray, and incompressible MHD flow past an exponentially accelerated semi-infinite vertical plate in presence of a transverse magnetic field. The medium of flow is porous. Arbitrary ramped temperature and diffusion thermo effects are also considered. Rosseland approximation method is used to describe the flux that

appears in the energy equation. The effects of different parameters on flow and transport characteristics are discussed with the help of suitable graphs. It is noticed that velocity field and concentration field decreases but temperature field increases with an upsurge in Schmidt number. Also, Nusselt number and skin friction rise with increasing chemical reaction parameter but lowers with increasing radiation parameter. Faster consumption of chemical substances decelerates both concentration and velocity but accelerates temperature of the fluid. An interesting outcome of our investigation is that both Dufour effect and arbitrary ramped temperature diminishes fluid velocity.

The objective of **Chapter VI** is to obtain an exact solution to the problem of a free convective, radiative, viscous, chemically reacting, heat absorbing, incompressible, and unsteady MHD flow past an exponentially accelerated moving vertical plate embedded in a porous medium. The fluid is assumed to be optically thick and non-gray. A magnetic field is applied in the transverse direction of the flow. Effects of arbitrary ramped temperature and thermal diffusion are also considered. Rosseland approximation method is used to describe the radiative heat flux that appears in the energy equation. Analytical solutions of the non-dimensional governing equations are obtained by adopting a closed-form of the Laplace transformation technique. The influence of various physical parameters on flow and transport characteristics is analyzed with suitable graphs. From the investigation, it is observed that increasing Soret number hikes both concentration and velocity fields. Ascending radiation parameter upsurges Nusselt number but declines Soret number.

Chapter VII deals with the effect of thermal diffusion in a free convective, radiative, viscous, chemically reacting, incompressible, and unsteady MHD flow past an exponentially accelerated moving inclined plate submerged in a porous medium. The fluid is taken as optically thick and non-gray. A uniform magnetic field is applied in the transverse direction of the plate. The ramped temperature effect is also considered. The radiative heat flux that appears in the energy equation is described by the Rosseland approximation method. A closed form of the Laplace transformation technique is adopted to obtain analytical solutions to the non-dimensional governing equations. A detailed discussion on the effects of various governing parameters on the velocity field, temperature field, concentration field, Nusselt number, Sherwood number, and skin friction are analyzed using suitable graphs and tables. The investigation shows that the Soret effect hikes both concentration and velocity of the fluid. The chemical reaction effect upsurges the process of mass transfer from the plate to the fluid.

Chapter VIII is directed to study the combined effects of induced magnetic field and ramped temperature in a chemically reacting, free convective, incompressible viscous and radiative, unsteady MHD flow past an exponentially accelerated moving vertical plate. The thermal diffusion effect is also taken into account. The fluid is assumed to be non-gray and optically thick. In the normal direction of the plate, a uniform magnetic field is applied. The radiative heat flux appearing in the energy equation is approximated by the Rosseland method. The Laplace transformation technique is used to find solutions to the non-dimensional governing equations. Effects of various governing parameters on velocity field, temperature field, concentration field, Nusselt number, Sherwood number, and skin friction are analysed using suitable graphs and tables. The present study reveals that high magnetic diffusivity hikes induced magnetic field. The thermal diffusion effect hikes the concentration field whereas the chemical reaction effect declines the velocity field. Radiation accelerates the rate of heat transfer but decelerates the rate of mass transfer from the plate to the fluid.