

7. (a) (i) Define reciprocal cone. Show that the cones  $ax^2 + by^2 + cz^2 = 0$  and  $\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c} = 0$  are reciprocal.

- (ii) Find the equation of the right circular cylinder whose guiding curve is  $x^2 + y^2 + z^2 = 9$ ,

$$x - y + z = 3.$$

$$5 + 5 = 10$$

- (b) (i) Find the equation of the director sphere to the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

- (ii) Show that from any point six normals can be drawn to a conicoid  $ax^2 + by^2 + cz^2 = 1$ .

$$5 + 5 = 10$$

The figures in the margin indicate full marks for the questions.

1. Answer **all** the questions :  $1 \times 10 = 10$

- (a) When the origin is shifted to a point on the x-axis without changing the direction of the axes, the equation of the line  $2x + 3y - 6 = 0$  takes the form  $lx + my = 0$ . What is the new origin ?

- (b) Find the centre of the ellipse

$$2x^2 + 3y^2 - 4x + 5y + 4 = 0.$$

- (c) Find the angle between the lines represented by the equation

$$x^2 + xy - 6y^2 = 0.$$

- (d) Transform the equation  $\frac{1}{r} = 1 + \cos\theta$  into cartesian form.

- (e) Find the equation of the tangent to the conic  $y^2 - xy - 2x^2 - 5y + x - 6 = 0$  at the point  $(1, -1)$ .

- (f) Express the non-symmetric form of equation of a line  $\frac{y}{p} + \frac{z}{c} = 1$ ,  $x = 0$  in symmetric form.

- (g) Write down the standard form of equation of a system of coaxial spheres.

- (h) Write down the equation of a cone whose vertex is origin and the guiding curve is  $ax^2 + by^2 + cz^2 = 1$ ,  $lx + my + nz = p$ .

- (i) Define a right circular cylinder.

- (i) Find the equation of the tangent plane to the conicoid

$$ax^2 + by^2 + cz^2 = 1$$

at the point  $(\alpha, \beta, \gamma)$  on it

2. Answer **all** the questions :  $2 \times 5 = 10$

- (a) If  $(at^2, 2at)$  is the one end of a focal chord of the parabola  $y^2 = 4ax$ , find the other end.

- (b) Show that the equation of the lines through the origin, each of which makes an angle  $\alpha$  to the line  $y = x$  is  $x^2 - 2xy \sec 2\alpha + y^2 = 0$ .

- (c) Find the point where the line

$$\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{4}$$

meets the plane  $x + y + z = 3$ .

- (d) Find the equation of the sphere passing the points  $(0, 0, 0)$ ,  $(a, 0, 0)$ ,  $(0, b, 0)$ ,  $(0, 0, c)$

(e) Find the equation of the plane which cuts the surface  
 $2x^2 - 3y^2 + 5z^2 = 1$  in a conic whose centre is  $(1, 2, 3)$ .

3. Answer **any four** questions :  $5 \times 4 = 20$

(a) Show that the equation of the tangent to the conic  $\frac{l}{r} = 1 + e \cos \theta$  at the point whose vertical angle is  $\alpha$  is given by

$$\frac{l}{r} = e \cos \theta + \cos(\theta - \alpha).$$

(b) Prove that the line  $lx + my = n$  is a

normal to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , if

$$\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2 - b^2)}{n^2}.$$

(c) Find the asymptotes of the hyperbola

$$2x^2 - 3xy - 2y^2 + 3x + y + 8 = 0$$

and derive the equations of the principal axes.

(d) Prove that the lines

$$\frac{x+5}{3} = \frac{y+4}{1} = \frac{z-7}{-2}$$

and  $3x + 2y + z - 2 = 0 = x - 3y + 2z - 13$  are coplanar. Find the equation of the plane in which they lie.

(e) The section of a cone whose guiding curve is the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, z = 0$  by the plane  $x = 0$ , is a rectangular hyperbola. Prove that the locus of the vertex is  $\frac{x^2}{a^2} + \frac{y^2 + z^2}{b^2} = 1$ .

(f) Find the centre and the radius of the circle

$$x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0,$$

$$x - 2y + 2z = 3.$$

Answer **either (a) or (b)** from the following questions :

10×4=40

4. (a) (i) Find the point of intersection of the lines represented by the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Contd.

(ii) Find the equation of the polar of the point  $(2, 3)$  with respect to the conic  $x^2 + 3xy + 4y^2 - 5x + 3 = 0$ .

5+5=10

(b) (i) Prove that the straight line  $y = mx + c$  touches the parabola

$$y^2 = 4a(x+a) \text{ if } c = ma + \frac{a}{m}.$$

(ii) Find the asymptotes of the hyperbola  $xy + ax + by = 0$ .

5+5=10

5. (a) Discuss the nature of the conic represented by

$$9x^2 - 24xy + 16y^2 - 18x - 101y + 19 = 0$$

and reduce it to canonical form.

(b) (i) Prove that the sum of the reciprocals of two perpendicular focal chords of a conic is constant.

(ii) Show that the semi-latus rectum of a conic is the harmonic mean between the segments of a focal chord.

5+5=10

Contd.

6. (a) (i) A variable plane makes intercepts on the co-ordinate axes, the sum of whose squares is a constant and is equal to  $k^2$ . Prove that the locus of the foot of the perpendicular from the origin to the plane is

$$(x^2 + y^2 + z^2)^2 (x^2 + y^2 + z^2) = k^2$$

(ii) Two spheres of radii  $r_1$  and  $r_2$  intersect orthogonally. Prove that the radius of the common circle is

$$\frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$$

5+5=10

(b) Show that the shortest distance between any two opposite edges of the tetrahedron formed by the planes  $x + z = 0, z + x = 0, x + y = 0, x + y + z = a$

is  $\frac{2a}{\sqrt{6}}$  and that the three lines of

shortest distance intersect at the point  $x = y = z = -a$ .

Contd.