*Total number of printed pages–7*

**3 (Sem–6/CBCS) MAT HE 4**

**2025**

**MATHEMATICS**

( Honours Elective )

Paper : MAT–HE–6046

***( Hydromechanics )***

*Full Marks : 80*

Time : Three hours

***The figures in the margin indicate full marks for the questions.***

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1. Answer the following questions : 1×10=10

   *(i)* Define specific heat of a body.

   *(ii)* Define surface of equal pressure.

   *(iii)* For an irrotational flow $curl\,\vec{q} \neq 0$. *(State True **or** False)*

   *(iv)* State Charle's law.

   *(v)* Name the instrument used to measure atmospheric pressure.

   *(vi)* What is tube of flow ?



*(vii)* What is the physical meaning of $curl\,\vec{q}$ ?

*(viii)* Define absolute zero of temperature.

*(ix)* Name the *two* methods of treating the general problem of Hydrodynamics.

*(x)* What is meant by rotational flow ?

2. Answer the following questions : 2×5=10

   *(a)* Show that the surface of equal pressure is intersected orthogonally by the lines of force.

   *(b)* If $\rho_0$ and $\rho$ be the densities of a gas at $0^0$ and $t^0$ centigrade respectively, then establish the relation $\rho_0 = \rho(1+\alpha t)$ where $\alpha = \frac{1}{273}$.

   *(c)* Establish the relation between local and individual time rate of change.

   *(d)* What is meant by velocity potential ? Does velocity potential exist for a rotational flow ?

   *(e)* Define streamline and path line of a fluid particle.



3. Answer ***any four*** of the following questions : 5×4=20

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   *(a)* A mass of fluid is at rest under the forces $X = (y+z)^2 - x^2$, $Y = (z+x)^2 - y^2$, $Z = (x+y)^2 - z^2$; find the density and the surfaces of equal pressure.

   *(b)* A fluid at rest is in equilibrium in the forces field $X = y^2 + z^2 - xy - zx$, $Y = z^2 + x^2 - yz - yx$, $Z = x^2 + y^2 - zx_zy$. Show that the curves of equal pressure and density are a set of circles.

   *(c)* If the absolute temperature $T$ at a height $z$ is a function of the pressures at two heights $z_1$ and $z_2$ then show that

$$\log\frac{p_2}{p_1} = -\frac{g}{R}\int_{z_1}^{z_2}\frac{dz}{f(z)}$$

   $R$ being the constant in the equation $p = R\rho T$.

(d) The velocity components of a flow in cylindrical polar coordinates are $\left(r^2 z\cos\theta, rz\sin\theta, z^2 t\right)$. Determine the components of the acceleration of a fluid particle.

(e) Test whether the motion specified by $\vec{q} = \frac{k^2(xi - yj)}{x^2 + y^2}$, $k$ is constant, is a possible motion for an incompressible fluid. If so, determine the equations of streamlines.

(f) The particles of a fluid motion move symmetrically in space with regard to a fixed centre; prove that the equation of continuity is $\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r} + \frac{\rho}{r^2}\frac{\partial}{\partial r}\left(ur^2\right) = 0$, where $u$ is the velocity at a distance $r$.

4. Answer ***any four*** parts : 10×4=40

(a) (i) Show that $\frac{x^2}{a^2}\tan^2 t + \frac{y^2}{b^2}\cot^2 t = 1$ is a possible form of bounding surface of a liquid.



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(ii) Two volumes $V_1$ and $V_2$ of different gases at pressures $P_1$ and $P_2$ and absolute temperatures $T_1$ and $T_2$ are mixed together, so that the volume of the mixture is $V$ and absolute temperature is $T$. Prove that the pressure of the mixture is $\frac{T}{V}\left(\frac{P_1V_1}{T_1} + \frac{P_2V_2}{T_2}\right)$.

(b) (i) A mass of homogeneous liquid, contained in a vessel, revolves uniformly about a vertical axis. Determine the pressure at any point and the surfaces of equal pressure.

(ii) Show that the centre of pressure of a circular area immersed in the liquid whose centre is at a depth $h$ below the surface, when the density of the liquid varies as the depth, is at a depth $\frac{2a^2h}{a^2 + 4h^2}$ below the centre of the circle, where $a$ is the radius of the circular area.



(c) (i) A liquid of given volume $V$ is at rest under the forces $X = \frac{\mu x}{a^2}, Y = -\frac{\mu y}{b^2}, Z = -\frac{\mu z}{c^2}$. Find the pressure at any point of the liquid and the surfaces of equal pressure.

(ii) For a perfect gas, establish the relation $C_p - C_v = R$, where the symbols have their usual meanings.

(d) Obtain Euler's equation of motion for a non-viscous fluid in the form $\frac{D\vec{q}}{Dt} = \vec{F} - \frac{1}{\rho}\vec{\nabla}P$.

(e) Obtain the equation of continuity for a fluid in motion in the form $\frac{\partial p}{\partial t} + div\left(P\vec{q}\right) = 0$ where $p$ and $\vec{q}$ are repectively the density and the velocity of the fluid. Deduce the form of equation of continuity when the fluid is homogeneous and incompressible.



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(f) Stream is rushing form a boiler through a conical pipe, the diameters of the ends of which are $D$ and $d$; if $V$ and $v$ be the corresponding velocities of the stream and if the motion be supposed to be that of divergence from the vertex of the cone, prove that $\frac{v}{V} = \frac{D^2}{d^2}e\frac{v^2 - V^2}{2k}$ where $k$ is the pressure divided by the density and suppose constant.

(g) Prove that if the forces per unit of mass at $(x, y, z)$ parallel to the axes are $y(a-z), x(a-z), xy$; the surface of equal pressure are hyperbolic paraboloid and the curves of equal pressure and density are rectangular hyperbolas.

(h) A hemispherical bowl is filled with water and two vertical planes are drawn through its central radius, cutting off a semilune of the surface, If $2\alpha$ be the angle between the planes, prove that the angle which the resultant pressure on the surface makes with the vertical is $\tan^{-1}\left(\frac{\sin\alpha}{\alpha}\right)$

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