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3 (Sem-6/CBCS) MAT HE 2

2025

MATHEMATICS

(Honours Elective)

Paper : MAT-HE-6026

(Biomathematics)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : 1×10=10

(a) The difference equation

$$x_{t+1} = ax_t + bt^2x_{t-1} + \sin(t) \text{ is a}$$

(i) non-autonomous

(ii) non-homogeneous

(iii) linear, provided a and b are constants

(iv) All of the above

(Choose the correct answer)

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(b) Let $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

Write the eigenvalues of the characteristic equation.

(c) Write the condition that a non-negative matrix A is a primitive.

(d) Why does a logistic growth often referred as a sigmoid growth ?

(e) The Juri condition or Juri test is satisfied for local asymptotic stability.

(Write true/false)

(f) Give an example of Malthusian growth.

(g) Write when a system is said to be persistent.

(h) Write a difference between continuous growth and discrete growth.

(i) Lyapunov stability is a technique that can be used to show _____ stability of an equilibrium. (Fill up the blank)

(j) Give an example of a simple SI model with no births and deaths.

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2. Answer the following questions : 2×5=10

(a) Write *two* basic requirements of a mathematical problem to be properly posed.

(b) Define a dominant eigenvalue.

(c) Check whether the matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & 0 & 0 \\ 0 & a_{32} & a_{33} \end{bmatrix}$$

satisfies Leslie matrix or not.

(d) Find the equilibria for the difference equation $x_{t+1} = ax_t^3$, $a > 0$.

(e) Write Routh-Hurwitz criteria for the given polynomial

$$P(\lambda) = \lambda^n + a_1\lambda^{n-1} + \dots + a_{n-1}\lambda + a_n,$$

where the coefficients a_i are real constants, $i = 1, 2, \dots, n$.

3. Answer **any four** questions : 5×4=20

(a) Find the general solution of the homogeneous difference equation

$$x_{t+2} - 16x_t = 0.$$

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- (b) Find the eigenvalues and then find the general solution to $X(t+1) = AV(t)$

$$\text{where } A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

- (c) Find all the equilibria for the difference equation

$$x_{t+1} = ax_t^3 + a_t, \quad a \neq 0.$$

Then determine whether they are locally asymptotically stable or unstable.

- (d) A mathematical model for the growth of a population is

$$\frac{dx}{dt} = \frac{2x^2}{1+x^4} - x = f(x), \quad x(0) \geq 0$$

where x is the population density. Find the equilibria and determine their stability.

- (e) With a suitable example, explain briefly phase plane analysis.

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The normalized form of a predator prey model has the form

$$x_{t+1} = (r+1)x_t - rx_t^2 - Cx_t y_t$$

$$y_{t+1} = Cx_t y_t + (1-d)y_t$$

where $r, c > 0$ and $0 < d < 1$.

Determine conditions for local stability of the equilibria.

Answer the following questions : $10 \times 4 = 40$

- (a) Discuss a Predator-Prey model with a suitable example by finding its equilibria, local stability. 10

Or

Discuss the behaviour of the system

$$\dot{x} = -ax + gx^2 - bxy$$

$$\dot{y} = -cy + dxy$$

where a, b, c, d and g are all constants.

- (b) Suppose the Leslie matrix is given by

$$L = \begin{bmatrix} 0 & \frac{3a^2}{2} & \frac{3a^3}{2} \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \end{bmatrix}, \quad a > 0$$

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- (i) Find the characteristic equation, eigenvalues and inherent net reproduction number R_0 of L .

- (ii) Show that L is primitive

- (iii) Find the stable age distribution. $2+3+5=10$

Or

Show that the solution to the pharmacokinetics model is

$$x(t) = \frac{1}{a} (1 - e^{-at})$$

$$y(t) = \frac{1}{t} + \frac{e^{-at}}{a-b} - \frac{ae^{-bt}}{b(a-b)} \quad 10$$

- (c) Discuss the system

$$\dot{x} = ax - gx^2 - bxy$$

$$\dot{y} = c'y - g'y^2 + dxy$$

in the phase plane, all the constants being positive. Will the predators die out if there is no prey? 10

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Or

A second order difference equation

$x_{t+2} + ax_{t+1} + bx_t = 0, a \neq 0$ has a characteristic polynomial with a root of multiplicity two, $\lambda_1 = \lambda_2 \neq 0$.

- (i) Show that $t\lambda_1^t$ is a solution of the difference equation.

- (ii) Show that the casoration $C(\lambda_1^t, t\lambda_1^t) \neq 0$ for any t . $5+5=10$

- (d) With a suitable example discuss briefly a SIR epidemic model. 10

Or

Discuss briefly about simple Kermack-McKendrick epidemic model.

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