Total number of printed pages–7

3 (Sem–6/CBCS) MAT HE 3

2025

MATHEMATICS

(Honours Elective)

Paper : MAT–HE–6036

(Mathematical Modelling)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×7=7

   (a) If R is the radius of convergence of any power series then what is the interval of convergence ?

   (b) Write Legendre's equation of order n.

   (c) Find the Laplace transform of $F(t) = e^{at}$ for $t \geq 0$.

   (d) State the linearity property of Laplace transform.

   (e) Find the value of $\Gamma\left(\frac{1}{2}\right)$.

   (f) The important step required for simulation approach in solving a problem is to

      (i) test and validate the model

      (ii) design the experiment

      (iii) conduct the experiment

      (iv) All of the above

   (g) What do you mean by Simulation Modelling ?

2. Answer the following questions : 2×4=8

   (a) Show that $x = 0$ is an ordinary point of

   $$(x^2 - 1)y'' + xy' - y = 0,$$

   but $x = 1$ is a regular singular point.

   (b) If the Laplace transform of the function $f(t)$ is given by $\frac{s+3}{(s+1)(s+2)}$, then find the value of $f(0)$.

   (c) Find the inverse Laplace transform of

   $$G(s) = \frac{1}{s^2(s-a)}.$$

   (d) Why is sensitivity analysis important in linear programming.

3. Answer ***any three*** questions of the following : 5×3=15

   (a) Find the power series solution of the differential equation

   $$(x^2 + 1)y'' + xy' - xy = 0.$$

(b) Find the exponents in the possible Frobenius series solution of the equation

$$2x^2(1+x)y'' + 3x(1+x)^3 y' - (1-x^2)y = 0.$$

(c) Evaluate : 2½+2½=5

(i) $L\left\{\frac{e^{-at} - e^{-bt}}{t}\right\}$

(ii) $L\left\{\int_0^t ue^{-u} \sin 4u \, du\right\}$

(d) Solve the following differential equation using Laplace transformation

$$(x-3)y' + 2y = 0.$$

(e) What is meant by random numbers in Monte Carlo simulation? Use linear congruence method to generate 8 random numbers using $a = 5$, $b = 1$ and $c = 8$ and taking initial seed $x_0 = 17$. 1+4=5

Answer ***any three*** questions of the following : 10×3=30

(a) Find the general solution in powers of x of the equation

$$(x^2 - 4)y'' + 3xy' + y = 0.$$

Then find the particular solution with $y(0) = 4$, $y'(0) = 1$.

(b) Find the Frobenius series solution of

$$2x^2y'' + 3xy' - (x^2 + 1)y = 0.$$

(c) (i) Solve the initial value problem $x'' + 4x = \sin 3t$, $x(0) = 0 = x'(0)$. 5

(ii) Find the inverse Laplace transformation to find $f(t)$ of the function

$$F(s) = \frac{s-2}{s^2 + 5s + 6}.$$ 5

(d) Using Monte Carlo simulation, write an algorithm to calculate the volume of the sphere

$$x^2 + y^2 + z^2 \le 1$$

that lies in the first octant $x > 0$, $y > 0$, $z > 0$.

(e) A small harbor has unloading facilities for ships. Only one ship can be unloaded at any time. The unloading time required for a ship depends on the type and amount of cargo and varies from 45 to 90 minutes.

Below is given a situation with 5 ships :

	Ship 1	Ship 2	Ship 3	Ship 4	Ship 5
Time between successive ships :	30	15	20	25	120
Unloading time :	40	35	60	45	75

(i) Draw the timeline diagram depicting clearly the situation for each ship, the idle time for the harbor and the waiting time.

(ii) Find the average waiting time for the ships. 8+2=10

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